

Each problem is a separate extra credit project. You may complete both projects but must turn them both in before the December 7th due date.

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Mrs. Hernandez comes across a gum ball machine one day when she is out with her twins. Of course, the twins each want a gum ball. What's more, they insist on being given gum balls of the same color. (They don't care what color they are, as long as they are the same color.)

Mrs. Hernandez can see that there are only white and red gum balls in the machine. The gum balls are a penny each and there is no way to tell which color will come out next. Mrs. Hernandez decides that she will keep putting pennies into the machine until she gets two gum balls of the same color.

What is the most that she might have to spend in order to satisfy her twins?

The next day, Mrs. Hernandez passes a different gum ball machine. This one has three colors — red, white, and blue. What is the most that Mrs. Hernandez might have to spend at this new gum ball machine in order to get matching gum balls for her twins?

Along comes Mr. Hodges with his triplets past the three-color gum ball machine. Of course, his children also insist that they all get the same color gum ball. What is the most that Mr. Hodges might have to spend?

The project

Solve the questions given above, then do some more examples of your own. Look for a way to organize the information and look for patterns. Your goal is to find a formula so that, if someone tells you the number of colors and the number of children, your formula will compute the maximum that the parent might need to spend. Write a report which explains how you derived your formula and how you can be certain that it will work in all situations. Your formula must be correct and your explanations must be clear and complete for full credit.

Who Will Win the Prize?

King Arthur was the king of Camelot. He loved inviting his knights over for parties around his round table. If there was something pleasant that the King could only give to one knight (an extra dessert, a dragon, the hand of his daughter in marriage, etc.), he had them play a game.

The game went like this:

First, King Arthur numbered the chairs around the table. At the start, a knight occupied every chair. (King Arthur did not sit at the table.) Then, he stood behind the knight in chair 1 and said, "You're in." Next, he moved to the knight in chair 2 and said, "You're out." That knight

¹Both problems are problems of the week from the *Interactive Mathematics Program*.

then left his seat and went off to stand at the side of the room to watch the rest of the game. Next King Arthur moved to the knight in chair 3 and said, "You're in." Then he said, "You're out" to the knight in chair 4, and that knight left his seat and went to the side of the room.

He continued around the table in this manner. When he came back around to the knight in chair 1, he said either "You're in" or "You're out," depending on what he said to the last knight. (If the last knight was "in," then the first knight would be "out," and vice versa.)

The King kept moving around the table, alternately saying "You're in" or "You're out" to the knights remaining at the table. If a chair was empty, he just skipped it. He continued until there was only a single knight remaining at the table. That knight was the winner.

The number of knights varied each week depending on who was sick, out chasing dragons, and so on.

The question

If you were the first one to sit down on any given night and you knew how many knights would be present that night, which chair number would you sit in to be certain that you would win the prize? Your solution should reliably predict the winning chair if you know the number of knights that will be present and should be faster to compute than the King's method. Use your method to predict which chair you should sit in if you know that 100 knights will be attending dinner? Write a report which explains how you derived your method and how you can be certain that it will work in all situations. Your formula must be correct and your explanations must be clear and complete for full credit.