Mth U180 **Type I and II error**

Definitions: A **Type I error** is where the null hypothesis is rejected when it is true.

A **Type II error** is where the null hypothesis is accepted when it is false.

The **power** of a test is 100-P(type II error).

Example 1: A company sells bottles that they claim have 16 ounces in them. They will admit that their claim is wrong (and refund any money), if a sample of 30 has a mean less than 15.8 ounces. If the standard deviation is 0.5 ounces:

a) find the probability of a type I error.

b) find the probability of a type II error and the power if the true mean is 15.9, 15.8, or 15.6.

ANSWER

(a) $H_0 : \mu = 16$

P(type I error) =P(reject H_0 when it's true)=P($\bar{x} < 15.8$ when $\mu = 16$)

$\Rightarrow z = \dfrac{15.8 - 16}{(.5/\sqrt{30})} = -2.2 \Rightarrow$ (from table) $97.22 \Rightarrow \frac{1}{2}(100 - 97.22) = \mathbf{1.39}\%$

(b) P(type II error)=P(accept H_0 when it's false)=P($\bar{x} > 15.8$ when $\mu = 15.9$)

$\Rightarrow z = \dfrac{15.8 - 15.9}{(.5/\sqrt{30})} = -1.1 \Rightarrow$ (from table) $72.87 \Rightarrow 50 + \frac{1}{2}(72.87) = \mathbf{86.435}\%$, the power is 13.565%

=P($\bar{x} > 15.8$ when $\mu = 15.8$) $\Rightarrow z = \dfrac{15.8 - 15.8}{(.5/\sqrt{30})} = 0 \Rightarrow \mathbf{50}\%$, the power is 50%

=P($\bar{x} > 15.8$ when $\mu = 15.6$) $\Rightarrow z = \dfrac{15.8 - 15.6}{(.5/\sqrt{30})} = 2.2 \Rightarrow$ (from table) $97.22 \Rightarrow \frac{1}{2}(100 - 97.22) = \mathbf{1.39}\%$, the power is 98.61%

NOTE: The probability of a Type II error and the power depend on the population mean. Since the population mean is unknown, the usual method is to find these for a series of possible population means (as is done in the above example) or to find it for a set difference that is 'important' (for example: If you are looking at a drug that lowers cholesterol, you probably would not care about a reduction of 1 mg/DL. You would decide what an 'important' reduction is.).

NOTE: The probability of a Type I error is set up to be the significance level of the test.

Example 2: The population mean of something is 40. A treatment is administered to a random sample of 50 and after $\bar{x} = 41.6$ and $s = 8$. Test at the 5% level to see if the population mean has increased.

a) Set up the null and alternate hypotheses and decide if you accept or reject the null.

b) Find the critical value in nonstandard form.

c) Give the P(type I error).

d) Give the P(type II error) if the mean is really 43 or 44 (use the given s).

e) Find the power of the test to notice an increase of 3 in the population mean.

ANSWER

(a) $H_0 : \mu = 40$ $H_a : \mu > 40$

z(test statistic)=$\dfrac{41.6 - 40}{8/\sqrt{50}} = 1.4$, critical value= (look up 100-2(5)=90% in normal table)=1.65.

Since $1.4 < 1.65$, accept H_0 and we accept that the population mean could be 40.

(b) Using $z = \dfrac{\bar{x}-\mu_0}{s/\sqrt{n}} \Rightarrow 1.65 = \dfrac{\bar{x}-40}{8/\sqrt{50}} \Rightarrow \bar{x} = 41.867$. **This means, H_0 will be rejected if $\bar{x} > 41.867$**

(c) 5%, as given

(d) =P(accept H_0 when it's false)=P($\bar{x} < 41.867$ when $\mu = 43$)

$\Rightarrow z = \dfrac{41.867-43}{(8/\sqrt{50})} = -1 \Rightarrow$ (from table) $68.27 \Rightarrow (100-68.27)/2 = \mathbf{15.865}\%$

=P(accept H_0 when it's false)=P($\bar{x} < 41.867$ when $\mu = 44$)

$\Rightarrow z = \dfrac{41.867-44}{(8/\sqrt{50})} = -1.9 \Rightarrow$ (from table) $94.26 \Rightarrow (100-94.26)/2 = \mathbf{2.87}\%$

(e)Power =1−P(type II error). We want P(type II error) if the mean is really 43 (an increase of 3), so power=$100-15.865 = \mathbf{84.135}\%$

Homework:

1. J claims that the mean is 9.4. He will admit he is wrong if the mean of a sample of 64 is more than 10. If the standard deviation is 1.8:

a) find the probability of a type I error.

b) find the probability of a type II error if the true mean is 9.8.

c) find the probability of a type II error if the true mean is 10.3.

2. K claims that the percent of people who will vote for him is 50% (or more). She will admit she is wrong if the percent from a sample of 800 is less than 46%.

a) Find the probability of a type I error.

b) Find the probability of a type II error if the true percent is 44

3. You will test at the 5% significance level to see if the mean is 50 or if it is more. If you use a sample of size 75 and know the standard deviation is 12:

a) find the critical value in nonstandard form.

b find the probability of a type I error.

c) find P(type II error) and the power if the mean is really 54 or 56.

4. A year ago the percent of people that agree with you is 40%. To see if things have changed, you take a random sample of 1000 and test at the 1% level of significance to see if the percent has increased.

a) Find the critical value in nonstandard form.

b) Find P(type II error) if the mean is really 42% or 47%.

c) Find the power of the test to find an increase of 7%.

5. You will test at the 5% significance level to see if the mean is 1650. If you use a sample of size 60 and know the standard deviation is 125:

a) find the critical value in nonstandard form.

b) find P(type II error) if the mean is really 1700.

6. A random sample of 240 finds $\bar{x} = 207$ with $s = 24$.

a) Test at the 5% level of significance if the mean could be 210 or if it is less.

b) Find the critical value in nonstandard form and use it to give a rejection test for H_0.

c) Find P(type II error) if the mean is really 202.

7. You are given $H_0 : p = 72\%$, $H_a : p > 72\%$, and n=250. If you do a hypothesis test at the 1% level:

a) find the critical value in nonstandard form and use it to give a rejection test for H_0.

b) find P(type II error) if p is really 80%.

8. You are given $H_0 : \mu = 36.4$, $H_a : \mu < 36.4$, and n=44. If you do a hypothesis test at the 2% level:

a) find the critical value in nonstandard form and use it to give a rejection test for H_0.

b) find P(type II error) if μ is really 40.